Governance of Clubs and Firms with Cultural Dimensions*

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October 1999

Abstract

The neoclassical way to cope with firms providing services, or with clubs procuring services, is restricted by the lack of institutional features. An institutional approach is introduced that requires a cooperative governance to realize the potential value-production by firms, or to realize the potential user-value by clubs. For each, a distinctive governance system is introduced. The firm requires an implementation governance to activate the value-production capacities of its service providers. It is empowered top-down by the unique top-position of the organization. The club, on the other hand, requires a representation governance to aggregate the user-values of its members for some common service and to order this service. It is empowered bottom-up by the service-users, i.e., the members of the club using that common service.

Institutional characteristics are also reflected in the distribution functions that are used in rewarding positions in firms and clubs. Some cultural dimensions are expressed in these distribution functions. That allows us to relate characteristics of governance systems to society's cultural dimensions.

Keywords: Governance, service economy, cooperative organization, club, firm, values, culture.

JEL-classification: C71, D23, D63, D7, H1, L22.

1 Introduction

Neoclassical economics has focussed on the external role of the firm in a competitive market environment. The firm was assumed to choose technological input-output combinations that maximize profits.

In the neoclassical tradition, the external interaction between traders in the general equilibrium model was anonymous and the internal interaction structure in a firm or a club was absent. A firm or a club, however, has an organization that is ruled by internal values and norms. It also has a mission or task that governs its internal activities. These activities have to be valued externally by its environment in order for the organization to survive, e.g., by the market system or the governmental system.

*Suggestions from Claude Menard, Myrna Wooders and Rob Gilles are gratefully acknowledged.

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The seminal paper introducing the cooperative nature of the firm has been written by Coase (1937). Coase observed that transaction costs are made by a firm to buy labor services on a market. When these services are demanded regularly on the external market, it may be cheaper for the firm to make these services available within the firm by means of long term contracts. The firm, as an organization of positions connected by agency relations, writes long term contracts for each position to be filled. The firm's organizational costs of writing and monitoring one long term contract may at most be equal to the organizational cost of writing and monitoring a number of short term contracts for the same position. Coase therefore introduced the organizational capacities of a firm as a production factor, distinguishable from the technological capacities embodied in the firm.

In order to determine the tradeoff between organizational and technological capacities, parameters need to be designed to describe organizational capacities. The Chief Executive Officer of a firm controls the performance of the firm by means of some of these organizational parameters, rather than the firm's technical parameters. For a hierarchically structured firm with a given production technology, Williamson (1967) has introduced the number of organization levels as the institutional parameter to control the profits of the firm. This organizational parameter has been investigated further by, a.o., Keren and Levhari (1979, 1982). These models are valid, however, for one specific technology, i.e., the linear production function, for one specific wage structure, i.e., proportional to the level in the hierarchy, and for homogeneous labor. They are therefore too restricted to determine the relation and the tradeoff between organizational and technological features, which is needed to describe the concept of a cooperative firm in a competitive environment. For a firm, these restrictions have been removed by van den Brink and Ruys (1996). This firm is placed in a competitive market environment in Ruys and van den Brink (1999).

Also consumption of a common service is typically a social activity. One way to describe the social nature of consumption is the concept of a public good, introduced by Samuelson (1954). The assumption that no consumer can be excluded from consuming such a good can be seen as an implicit description of anonymous social interaction in consumption. The second assumption, however, non rivalry in consumption, excludes any form of personal interaction in consumption. The concept of a local public good introduced by Tiebout (1956) improves upon Samuelson's concept by restricting the set of consumers involved, so a consumer may choose not to enter the local public good if rivalry or harmful interaction diminishes her utility too much. The net effect of internal interaction is beneficial for all involved. The Tiebout tradition has focussed on clubs as political jurisdictions, allowing for a partition of the population as part of the basic description of the economy. Such a partition will result also in our approach. Ellickson et al. (1999) built on the Buchanan tradition in which many types of clubs are possible and people may be member of more clubs. They view the activity of a club as a public project (see Mas-Colell, 1980) rather than as provision of some level of a public good. A club membership is an opening in a club available to agents with specified characteristics[1]. Agents choose both private goods and club memberships, and private goods and club memberships are treated and priced in parallel fashion.

[1]In Cornes and Sandler (1986), a club is described as a voluntary group deriving mutual benefit from sharing one or more of the following: production costs, the members' characteristics, or a good characterized by excludable benefits. These goods were previously called club goods. Cornes and Sandler's definition broadens clubs to include more than collectives sharing excludable public goods, which is in line with our definition. Our definition focusses on the empowerment or agency relation: in a club bottom up, in a firm top down. Membership is voluntary and implies carrying a burden.

Another paper building on the notion that agents may belong to multiple clubs is Shubik and Wooders (1982). In some respects this work is more compatible with ours than Ellickson et al, since Shubik and Wooders allow arbitrary club structures, and increasing returns to club size. They show that approximate cores are nonempty and treat similar agents similarly. Unlike other recent research on economies with clubs, we consider agents that are homogeneous except for a location parameter. Agents interact in this parameter in a typical way, typical for the common service considered. Agents at different locations are similar, if at these locations the interaction configurations with other agents are identical.

In this paper we discuss a model for the provision and for the procurement of a *common service* to society. The provision of the service is carried out by a (public) firm. In van den Brink and Ruys (1996), the internal organization of a private firm is described by (i) a *value-production function* defined on a set of service providers, describing the technical production possibilities of the firm, (ii) a governance consisting of positions within the firm connected by agency relations, (iii) an internal cost parameter, and (iv) a *remuneration system* determining the internal wages and profit in the firm. The value-production function describes the effects of cooperation between service providers in the value-production. However, the service-providers can only render services to the clients and, by being paid for these services, create the value-added of the firm, if they are supported by an organization. Expanding the governance of the firm, i.e., increasing the number of levels in the hierarchical firm structure, enhances the value of the service rendered by the front-positions (such as a surgical team) to the client. The higher levels in the organization of the firm exist only to coordinate and to improve the productivity of the front-positions. Such a governance is called an *implementation governance*. The remuneration of each position in the organization is determined by a remuneration system that distributes the value-added of the firm over all positions in the firm, including profit assigned to the top-position. The actual number of levels is determined by the profit maximizing behavior of the governor in the top-position.

The same approach can be followed for describing the cooperative nature of consumption. For that purpose the concept of a club is used. We built on the Ellickson e.a. approach and give these clubs an internal organization. A set of these clubs that procure all members of society with the same common service is called a polity of the society. It is described by (i) a *user-value function* defined on the set of service-receivers, called members of the society, describing the willingness-to-pay of various groups of members for obtaining the common service, (ii) a governance consisting of positions within the polity connected by agency relations, and (iii) a *budget allocation function* assigning budgets to each position in a polity of clubs in order to provide the officers with the means to procure the service. An important difference between the implementation governance of a firm and the governance of a club is that the authority within a club is not allocated top-down, but bottom-up. In contrast to the firm, the members in the front-positions of the polity constitute the highest authority; they delegate power to higher levels within the organization. This type of governance is called a representation governance. The degree of governance within a polity is again determined by the costs and benefits of organizing.

The concept of an implementation and representation governance is rich enough to describe other organizational parameters besides the number of levels. In his seminal work, Hofstede (1980), on the basis of a large scale survey in more than 40 countries, identified some dimensions of culture that characterize a society, viz. its governance structures. The model described in this paper allows for defining precisely these cultural dimensions in terms of governance structures. Here we relate these cultural dimensions to the implementation and representation governance of a common service. We concentrate on two of these cultural dimensions. *Uncertainty avoidance* refers to the extent to which people feel threatened by uncertainty and ambiguity and try to avoid these situations. *Collectivism-individualism* reflects whether people look after themselves and their immediate family only or belong to 'in-groups' which 'look after them' in exchange for loyalty. We focus on the role of the distribution function that can be used as remuneration sytem in a firm or as budget allocation function in a club. We discuss a class of distribution functions that describe part of the internal rules and norms of society. Using these distibution functions we illustrate how different degrees of complementarity in value-production and user-value influence the size of firms and clubs. In that way we have related the implementation and representation governance of a common service to the cultural dimensions mentioned above.

This paper is organized as follows. In Section 2 we describe a polity of clubs that is associated with the procurement of a common service. In Section 3 we discuss firms associated with the provision of the common service. In Section 4 we study a class of distribution functions and analyze their effect on the representation and implementation governance of a common service. Finally, in Section 5 we relate these effects to cultural dimensions.

2 The procurement of a common service

The procurement of a service by a group of people in a society is organized by means of a set of clubs forming together a polity for that common service. The club members voluntarily form a club, i.e., a group deriving mutual benefits from sharing a common service. We first describe some characteristics of the user-value function of a common service. After that the governance of a club is analyzed.

2.1 The user-value function of a common service

The concept of a common service generalizes upon the commodity concept. A tradeable commodity is a carrier of some desirable property. An agent enters into an exchange transaction with another agent if the commodity that the other agent owns carries properties that she prefers over the properties carried by her commodity. The user-value of a commodity is assumed to be independent from the seller of that commodity. The user-value of the car you own is independent from the person who sold you the car. In the case of a service, however, the relation between the provider (seller) and the receiver (buyer) becomes crucial. The service rendered by a hairstylist involves the buyer personally, as does the service of a medical doctor. So the provider of a service gets access to the receiver of the service in order to provide the property desired by the receiver. A **service** is thus a relation between the provider and the receiver, which relation carries over the desired property. The carrier is the relation itself. When that relation is voluntary and anonymous, meaning that any provider or receiver may be substituted in the relation by another (identifiable) provider or receiver, it is called a *standard service*. Nonstandard services are person-specific services that cannot be provided by standard economic transactions, such as cases in which the receiver is vulnerable to the supplier. Hairstylist or medical services are usually standard services, because these services are embedded in a legal framework that provides protection against involuntary involvements.

Since a service is carried by a relation, we need to specify the location of the nodes of this relation, i.e., the service provider and the service receiver. In this paper we assume that the providers as well as the receivers are ordered linearly and are located on the line of integers. A service received is, for example, a (specific) student receiving a lesson from a (specific) teacher, or a (specific) patient receiving treatment from a (specific) medical doctor. The **user-value** of a service represents the net benefits of this service for a specific receiver or user. Interaction between users may increase or decrease that value: a student may learn more in a class of interacting students, or a patient may recover slower in a room crowded with patients.

The scope of interaction between the receivers of a standard service eventually determines a hierarchical structure in the service. The education service, e.g., may have several levels: the student as individual receiver, the interacting students on the level of a class, the interacting students on the level of the school, et cetera. On each level a specific educational service may be offered as a segment of the aggregated education service. This hierarchical structure of the service partitions the set of students into subsets. If a standard service can be decomposed hierarchically in levels of aggregation of interacting users or receivers, it is called a **common service**. At each level of user-interaction, *segments* of that common service can be specified, where a segment is determined by the size of the group interacting on that level[2]. These segments of a common service are at the basis of the hierarchy in the polity procuring this common service.

In the model we consider a common service to be provided to a finite set of m users or *members*. The set of all members is called the *society* for that common service and is denoted by N_0. We assume the society N_0 to be a lineraly ordered finite set of m members denoted by $N_0 = \{i_{0,1}, \ldots, i_{0,m}\}$. The **user-value function** of a common service is a function $u\colon 2^{N_0} \to \mathbb{R}$ defined on the power set of N_0, which assigns to all possible coalitions $E \subset N_0$ of members the user-value of the common service, *if* this service is provided to these members. This function is **homogeneous** with respect to the users of the common service, meaning that any service receiver can replace another service receiver in the domain of the function without affecting the user-value[3]. We also assume the user-value function to be **monotonic** implying that $u(E) \leq u(F)$ if $E \subset F \subset N_0$.

Note that the user-value function u is independent of the governance. The various degrees of interaction between its users determine, however, the segments of the common service and eventually the number of levels in the organization's hierarchy. In order to realize potential user-value, a representation governance is required that makes it possible for the members to decide effectively, to procure the desired level of the common service, and to reap the fruits of cooperation. This representation governance is modeled by a polity of clubs for the common service.

[2]This ordering of segments of a common service corresponds with the subsidiarity principle.

[3]So in the case of a homogeneous user-value function of a service, the service receivers are identical with respect to their consumption abilities regarding this service, but not as members of a society in which a consumers' organization is formed. Our approach is also suited for heterogeneous consumers, which however complicates the results.

2.2 The governance required for procuring a common service

For a given system of budget allocation, the interdependence between the user-value function of a common service and its representation governance, viz. the size of the organization, is shown in this section. The representation governance for a common service consists of a structure of clubs, called the polity of that service. A **club** is a voluntary group deriving mutual benefits from procuring and sharing a common service characterized by excludable benefits, by establishing an organization with a representation governance that is empowered by the members. Examples of a club are: the legislative branch of the government in a society, the policy making branch of a union, of an association, or of a cooperative, a household, all as far as it concerns the collective decision making in that organization to buy a common service, or to empower the executive branch of the organization to provide it. For the government of a country, the spatial characteristic determining the segments of a common service represents the territorial subdivision of the nation into states, provinces, counties, cities, et cetera.

The user-value function defined in the previous subsection describes the potential benefits users may receive from the procurement of a common service. These potential benefits can only be realized by means of an organization, which is formally described as follows. The members of a society are coordinated by a **representation governance** which is represented by a directed graph, G_n, of positions and agency relations, where the number of positions from the top-position to a bottom-position is equal to n. In a representation governance the direction of the arrows, from principal to agent, is bottom-up. The member-positions are denoted by $L_0 := N_0$. The other positions are called *officer-positions* with the *top-position*to be occupied by the commissioners of the governance. The governance with n levels is denoted by (N_n, G_n) with N_n being the set of (member and officer) positions and G_n being the agency relations between these positions. If $N_n = N_0 = L_0$ then all members are treated as independent decision units with no governance between them (each member is its own officer). The corresponding governance structure is given by $G_n(i) = G_0(i) = \emptyset$ for all $i \in N_0$, and thus contains no relations. In order to establish procurement of the service to more than one member the governance must be expanded with officer positions and agency relations. We assume that the **scope of user-interaction** s of the common service is uniform, i.e., each first level officer position is controlled and empowered by the same number s of members, and each other officer position is controlled by the same number of officer positions one level below. The uniform scope of user-interaction is only assumed for reasons of simplicity and is not essential for our approach. We also assume that the scope of user-interaction s, which is derived from the user-value function, is given. A one level governance makes it possible for the sets $\{i_{0,1}, \ldots, i_{0,s}\}$, $\{i_{0,s+1}, \ldots, i_{0,2s}\}$, ..., $\{i_{0,m-s+1}, \ldots, i_{0,m}\}$ to coordinate decision making within these sets[4], and thus to procure the common service for these sets. However, this requires the presence of level one officer positions $L_1 = \{i_{1,1}, \ldots, i_{1,m/s}\}$, and thus $N_1 = L_0 \cup L_1$ forms the set of positions in a one-level polity. The governance structure G_1 is defined conformly by $G_1(i_{1,k}) = \emptyset$ for $k \in \{1, \ldots, m/s\}$, and $G_1(i_{0,(k-1)s+1}) = \ldots = G_1(i_{0,ks}) = i_{1,k}$ for all $k \in \{1, \ldots, m/s\}$. Expanding governance further yields governance levels $L_n = \{i_{n,1}, \ldots, i_{n,m/s^n}\}$ with $N_n = \bigcup_{l=0}^{n} L_n$. The maximal number of governance levels is equal to $n_{max} = {}^s\log m$ and $L_{n_{max}} = \{i_{n_{max},1}\}$. See Figure 1 for a one level and two level representation governance with $m = 4$ front positions and scope of user-interaction $s = 2$.

Since the structure G_n is empowered bottom up it has a forest structure, called the *polity*

[4] For simplicity we assume that there exists a number $k \in \mathbb{N}$ such that $m = ks$. This is not essential for interpreting our results.

of a service, which may consist of various tree structures refered to as *clubs* in the polity. A consequence of the bottom up empowerment is that the roles on the agency relations are such that the principal on a relation is the one that is closest to the members, and the agent is the one that is closest to the top. So, we refer to $i \in N_n \setminus N_0$ as the *agent* on the relation with $j \in G_n(i)$, and j is called the *principal* on that relation. A position having no agents then belongs to the set of top-positions, L_n, to be occupied by the commisioners in an n-level polity, while the member-positions are the positions with no principal. The other positions are intermediate officer positions. By $N_0 = L_0$ being a finite set of members it readily follows that the number of levels n is finite.

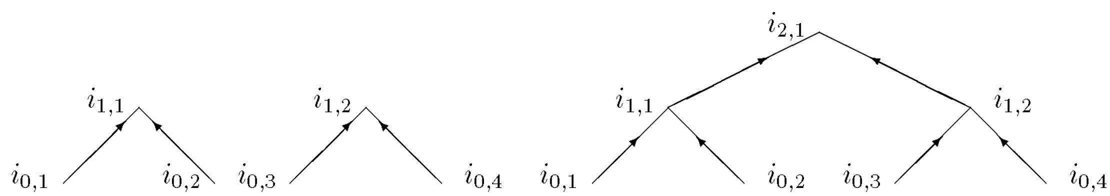

Figure 1: A one-level and a two-level representation governance for a society with $m = 4$ and scope of user-interaction $s = 2$

Given governance level n we define the partition $\mathcal{P}^n = \{P_1^n, \ldots, P_{m/s^n}^n\}$ with $P_k^n = \{i_{0,(k-1)s^n+1}, \ldots, i_{0,ks^n}\}$, $k \in \{1, \ldots, m/s^n\}$, to be the partition of N_0 into maximally connected subsets in G_n. For the examples illustrated in Figure 1 these partitions are $\mathcal{P}^1 = \{\{i_{0,1}, i_{0,2}\}, \{i_{0,3}, i_{0,4}\}\}$ and $\mathcal{P}^2 = \{N_0\}$, while $\mathcal{P}^0 = \{\{i_{0,1}\}, \{i_{0,2}\}, \{i_{0,3}\}, \{i_{0,4}\}\}$. In this example, if $n = 1$ then clearly the members cannot be treated anonymous. This is also reflected in the *value taxed* with governance level n which is defined by

$$v^n(E) = \sum_{P \in \mathcal{P}^n} u(E \cap P), \text{ for all } E \subset N_0.$$

Finally, for each level n, the contributions and resources from the members are allocated according to a **budget-allocation function**, β. This function assigns a budget $\beta_i(u, n)$ to each position $i \in N_n$ in the polity in order to provide its officers with the means to procure the corresponding segment of the common service. The budget-allocation function is **budget neutral** if the budget allocation function exactly distributes total user value over all positions, i.e., $\sum_{i \in N_n} \beta_i(u, n) = u(N_0)$. In game theory this property is called *efficiency*. The budget-allocation function is **symmetric** if, for a homogeneous user-value function, each position within one level is assgned the same budget. In that case we can denote the budget assigned to each position in level ℓ of a polity with n governance levels by $\beta_\ell(u, n)$, i.e., $\beta_\ell(u, n) = \beta_i(u, n)$ for all $i \in L_\ell$. Then $\beta_0(u, n)$ denotes the budget assigned to each front-position or member of a club, and $\beta_n(u, n)$ denotes the budget assigned to each top-position or commissioner of a club.

Definition 2.1 *An* **n-level polity***, denoted by $C_n = (u, N_n, G_n, \beta)$, consists of (i) a user-value function u, defined on the set of members N_0, being homogeneous and monotonic, (ii)* *a representation governance structure (N_n, G_n) with N_n being a non-empty and finite set of positions and G_n being a set of agency relations between these positions having a forest structure with constant scope of user-interaction, and (iii) a budget-allocation function β. Each connected component in a polity is called a* **club**.

The set of all n-level polities is denoted by $\mathcal{C}_n$. The parameter n is a variable for the polity. Therefore, a description of a polity should contain for every $n \in \mathbb{N}$ how the clubs looks like. We introduce a *polity mapping* as a mapping which assigns an n-level polity to every $n \in \mathbb{N}$. The set of positions differs for different $n \in \mathbb{N}$. However the set of members $L_0 = N_0$ is the same for every size.

Definition 2.2 *A* **polity mapping** *is a function $C: \mathbb{N} \to \bigcup_{n\in\mathbb{N}} \mathcal{C}_n$ such that $C(n) \in \mathcal{C}_n$.*

The members, forming a society for a given common service, determine both the number of clubs in the polity structure and their level, n, so as to maximize their net user-value as reflected in the budgets assigned by β. The optimal size of the polity is determined by the members. They maximize the budgets assigned to the positions in N_0. Their decision rule is formulated as follows.

Definition 2.3 *The* **optimal polity level** *is the lowest level of the polity that maximizes the budgets allocated to the members in N_0.*

In Section 4 we discuss some particular budget-allocation functions. Here we discuss a result for all budget-allocation functions satisfying budget neutrality, symmetry and the property that all budgets are nonnegative and that a positive budget is assigned to each position in a governance that generates a positive user-value, i.e., for every polity size n, every monotone user-value function u and every $i \in N_n$, it holds that $\beta_i(u, n) \geq 0$, with strict inequality if $v^n(N_0) > 0$.

An extreme user-value function is the linear user-value function $u(E) = c|E|$, $c > 0$, for all $E \subset N_0$ in which user-value is *separable*. Another extreme is a complementary user-value function given by $u(N_0) = c|N_0|$, $c > 0$, and $u(E) = 0$ if $E \neq N_0$, in which each member is necessry in order to generate a positive user-value. (In Section 4 also discuss intermediate cases for particular budget-allocation functions.)

Proposition 2.4 *Suppose that the budget-allocation function β satisfies budget neutrality, symmetry and assigns non-negative budgets to all positions if user-value u is monotone, and positive budgets if $v^n(N_0)$ is positive. If the user-value function is linear then the optimal polity size n^* equals zero, and the polity consists of m clubs. If the user-value function is complementary then the optimal polity size n^* equals the maximal size n_{max}, and the polity consists of one club.*

Proof

If user-value is linear then by budget neutrality and symmetry we have that $\beta_0(u, 0) = c$. Since $v^n(N_0) > 0$ it must hold that $\beta_l(u, n) > 0$ for all $n \in \{1, \ldots, n_{max}\}$, $l \in \{0, \ldots, n\}$. Budget neutrality then implies that $\beta_0(u, n) < c$ for all $n \in \{1, \ldots, n_{max}\}$. So, the optimal polity level is $n^* = 0$.

If user-value is complementary then by budget neutrality, symmetry and the fact that $\beta_l(u, n) \geq 0$ for all $n \in \{1, \ldots, n_{max}\}$, $l \in \{0, \ldots, n\}$, we have that $\beta_0(u, n) = 0$ if $n < n_{max}$.

Since $v^{n_{max}}(N_0) > 0$ it must hold that $\beta_0(u, n_{max}) > 0$. So, the optimal polity level is $n^* = n_{max}$.

$\square$

If user-value is linear, members do not want to pay taxes to invest in a representation governance because governance cannot improve user-value, while it induces the cost of the officer positions. If user-value is complementary then full governance is necessary in order to generate a positive user-value, and thus the maximal number of governance levels will be formed. For intermediate cases governance levels between 0 and n_{max} can be possible.

3 The organization of labor as a common service to the firm

The main problem for a firm that provides a service or produces commodities, is to determine the extent of the production of the firm and the number of production levels within the firm to support this provision, i.e., the optimal degree of organization. This is determined endogenously by the structure of the value-production function, representing the productive interaction between the providers of this service, by the remuneration system applied within the firm, and by the external prices of inputs and outputs. That problem is faced in this section.

It has already been remarked that the internal organization of the firm is not considered in neoclassical economic theory. The firm is simply an entity that transforms some commodities into other commodities. In his seminal work Williamson (1967) investigated the issue of an optimal size of a hierarchically organized firm and introduced the internal organization of the firm in economic analysis. Several authors, in particular Keren and Levhari (1979, 1982) extended Williamson's work. Our approach to the firm builds on this tradition, but generalizes it by allowing for different production functions and remuneration systems. We take authority relations into account but also other relations within the organization.

The second difference is that our approach is suited to immaterial services rather than to material commodities. The production function is an appropriate tool to represent the relation between quantitative inputs and outputs. This tool is less appropriate for the case of services and for describing behavioral features of the individual agents concerned. The concept of a common service has been introduced in the previous section and generalizes upon the commodity concept. Both a commodity and a common service may be the output of the production process described in this section. It is assumed that the value of this output is exogenously determined by markets, as are the values of the inputs of the firm, viz. labor and capital. However, we consider the input of labor as a common service for the firm, which labor generates the value added of the firm by providing some particular service to external clients or by producing specific commodities for them. The subset of labor that directly provides this service is called the set of *service-providers* and denoted by W_m, where m indicates the size of this set. The value-production capabilities of the service providers are expressed by the **value-production function** $f_m : 2^{W_m} \to \mathbb{R}$. These capabilities, however, can only be realized if there exists an organization supporting the service-providers, i.e., the governance. For reasons of simplicity, we assume in this paper that the service-providers in W_m are *homogeneous*, meaning that their individual labor-service capabilities are identical and that these capabilities differ only by their degree of cooperation. It follows that the value production can be represented by a **homogeneous** value-producetion function[5] $f_m: \{1, \ldots, |W_m|\} \to \mathbb{R}$.

So labor input can only be active by assuming some position or role in the governance of the firm. The service-providers occupy the front-worker positions on the lowest level of the organization of the firm. Since firms of different size have different sets of front-worker positions to be occupied by workers, the domain of production is different for different firms.

The **implementation governance** of a firm is a pair (N_n, G_n) with N_n a finite set of *positions*, and G_n a set of *agency relations*, both parametrized by an index, $n \in \mathbb{N}$. It is assumed that there is a unique position having no principal, which is called the *top position*, denoted by i_0, and which will be occupied by the *governor* of the firm. It also is assumed that each other agent has one principal, so there is no cycle in the governance structure. It follows that the governance structure (N_n, G_n) is a directed graph with a tree structure, its root being the top-position i_0, and the end-points forming a nonempty set of positions having no agents. This is the set of *front-worker positions* in the firm, denoted by $W_n = \{i \in N_n \mid G_n(i) = \emptyset\}$, where the service-providers are active[6]. A difference with the representation governance of a polity is the fact that on the relation (i, j), $j \in G_n(i)$, the principal i is the position closest to the top, while the position closest to the front-positions is the agent j. Another important difference with the representation governance of a polity is that the set of front-worker positions in a firm is variable and depends on the number of levels, while the set of members occupying the front positions in a polity is fixed. This is also reflected in the fact that the production technology of the firm, which is defined on the front-worker positions, has a variable domain. The labor input of the production process is carried out by the *workers* occupying these front-worker positions. The other positions, $N_n \setminus (W_n \cup \{i_0\})$, are the *intermediate positions* which serve to increase the productivity of the front-worker positions.

Since the implementation governance structure has a tree structure, a *level* in the organization can be defined as the set of positions each of which has the same distance to the top-position. Let $L_0 = \{i_0\}$ represent the top-level with the governor-position of the firm. The l^{th} level in an n-level firm then, recusively, is defined by $L_\ell = \bigcup_{i \in L_{\ell-1}} G_n(i)$, for $\ell = 1, \ldots, n$.

Additional structure is required to guarantee that L_n is equal to the set of positions having no agent. This is obtained by first assuming that each principal in the firm has the same number, s, of agents, called the *span of control*[7]. So $|G_n(i)| = s$ for all $i \in M_n = N_n \setminus W_n$. The positions at the *$\ell$-th level* of the firm belong to the set $L_\ell = \{i_{\ell,1}, \ldots, i_{\ell,s^\ell}\}$, for $\ell \in \{1, \ldots, n\}$. The relational structure is then specified by $G_n(i_{l,k}) = \{i_{l+1,(k-1)s+1}, \ldots, i_{l+1,ks}\}$, for $l \in \{0, \ldots, n-1\}$ and $k \in \{1, \ldots, s^l\}$, and $G_n(i_{n,k}) = \emptyset$ for $k \in \{1, \ldots, s^n\}$. It may be noticed that $i_{0,1}$ denotes here the top position i_0. Clearly, the set of front-worker positions, W_n, equals L_n and the set of principal-positions, M_n, is equal to $N_n \setminus L_n = \bigcup_{\ell=0}^{n-1} L_\ell$.

So the implementation governance (N_n, G_n) can be characterized by a parameter, viz. the number of levels n. In Figure 2 the governance structure of a one-level and two-level firm

[5] Alternatively, a *heterogeneous* production process is represented by a value-production function $f_{W_m}: \{0,1\}^{|W_m|} \to \mathbb{R}$, where W_m denotes the set of m service-providers who differ in individual labor-service capabilities. This affects the governance, of course. For reasons of simplicity we will only consider homogeneous labor-service capabilities in this paper.

[6] We denote the set of front-worker positions in this firm by W_n for notational convenience, although n need not be equal to the number of front-worker positions in this set.

[7] This assumption is not essential for our approach. It is, in fact, an assumption on the value-production function that generates a specific, simple interaction structure. In principle each level may have a different span of control, also in the case of a homogeneous value-production function.

is given for the case that the span of control, s, equals 2.

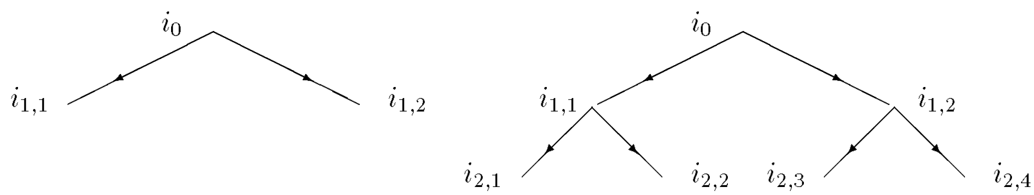

Figure 2: A one-level and a two-level implementation governance for a firm with span of control 2

The number of positions in the firm, $|N_n|$, equals $\sum_{\ell=0}^{n} s^\ell = (s^{n+1}-1)/(s-1)$, the number of principal positions, $|M_n|$, equals $\sum_{\ell=0}^{n-1} s^\ell = (s^n-1)/(s-1)$, and the number of front-worker positions, $|W_n|$, is equal to s^n in an n-level firm.

The production process in an n-level firm is described by the homogeneous value-production function $f_{s^n}\colon \{1,\ldots,s^n\} \to \mathbb{R}$. Homogeneous labor inputs require a homogeneous governance structure. So the employee positions within one particulal level should be identical in the governance structure, which is the case in the structure defined above.

An important difference with the representation governance of a polity discussed in the previous section is the fact that the implementation governance of a firm is expanded top-down (implying there is always one top position with varying number of front positions), while the representation governance of the polity is expanded bottom-up (with fixed set of front positions, called members, and varying number of top positions). The 'scope of production' in the firm structure, represented by the production function f_{s^n}, therefore depends on the number of firm levels while the 'scope of tax burden' in the polity is independent of the size of club structure, i.e., the user-value function u is independent of n.

The sequence of agency relations decentralizes decision making at each consecutive level and allows to decrease the complexity of the decision problem at each level. It results, however, in certain level-dependent agency costs. These *agency costs* are stated as a percentage of final production and are represented by $1-\alpha^n$, with the parameter α between zero and one. They therefore are increasing in the number of hierarchical levels. Examples of such costs are the facilities needed for the coordinators to operate, resulting in a loss of output, or costs involved in the processing and control of level-dependent budgets and information, implying a loss of control of a coordinator over the behavior of his successors. Adding a level in the organization may thus benefit the governor by increasing the scale of production, at the cost of an increase in agency costs.

We assume the value-production function to be *monotonic* implying that $f_{s^n}(|E|) \leq f_{s^n}(|F|)$ if $E \subset F \subset W_n$. An important subclass of monotonic value-production functions is the class of value-production functions that are monotonic and *supermodular* satisfying $f_{s^n}(|E|)+f_{s^n}(|F|) \leq f_{s^n}(|E\cup F|)+f_{s^n}(|E\cap F|)$ for all $E,F \subset N$. Clearly, supermodular value-production functions may exhibit increasing returns in the sense that they favor producing with larger sets of front-worker positions. Since the span of control of the firm is assumed to be given, the only way to increase the number of front-worker positions is to increase the number of levels in the firm. An extra level has a positive effect on value added through the monotonic value-production function. On the other hand, there is the negative effect of the level dependent agency cost.

If all front-worker positions are effectively occupied then a gross revenue equal to $f_{s^n}(s^n)$ is generated[8]. Net revenue or *value added* is obtained by subtracting the level-dependent cost from this gross revenue[9] yielding $\alpha^n f_{s^n}(s^n)$. Note that the parameter α, being the complement of the level-dependent agency cost parameter, can be seen as an *agency efficiency* parameter. It may correlate with the span of control parameter s, but both are given here.

Another important difference between the implementation and representation governance is reflected in the definition of value added. Positions within one level of the firm are treated anonymous if the labor inputs are homogeneous. Because of the tree structure of the implemetation governance the front-workers positions are always connected. In the representation governance of a polity the members cannot be treated anonymous because a particular member can be connected to some members while not connected to other members. This is reflected in the partition $\mathcal{P}^n$, and in the definition of value taxed of the polity discussed in the previous section.

By definition the value added of a firm also equals the reward paid to the production factors, i.e., the value added equals the sum of the positional wages and the positional returns on capital. The value added of a firm is distributed among all positions in the firm according to some **remuneration system** being a function φ which assigns a distribution of value added over the positions in the firm to every value-production function f_{s^n} with governance structure (N_n, G_n) and level-dependent agency cost $1 - \alpha^n$. The income allocated to some position in the firm is called a *positional income*. Since the implementation governance structure and level dependent agency cost are determined by n, we denote the positional income allocated to position $i \in N_n$ in a firm producing according to f_{s^n} by $\varphi_i(f_{s^n})$. We assume the remuneration system to satisfy budget neutrality, symmetry and structural monotonicity. Structural monotonicity in a firm means that a supervisor does not receive a lower wage than his successors[10], i.e., for every firm size n and every monotone value-production function f_{s^n} it holds that $\varphi_i(f_{s^n}) \geq \varphi_j(f_{s^n})$ for all $i \in M_n$ and $j \in G_n(i)$. This remuneration system determines the *positional wages* that eventually are paid to the laborers occupying the positions. Since we assume a homogeneous firm with a symmetric remuneration system we can speak about wages assigned to levels instead of wages assigned to positions, i.e., $\varphi_\ell(f_{s^n}) = \varphi_i(f_{s^n})$ for all $i \in L_\ell$ is the wage assigned to positions in level $\ell \in \{1, \ldots, n\}$. Similarly, the positional return for the top position is called the *net value added* or profit of the firm and is denoted by $\varphi_0(f_{s^n})$.

Definition 3.1 *An* **n-level firm***, denoted by $F_n = (f_{s^n}, N_n, G_n, \alpha, \varphi)$, consists of (i) a set of value-production functions f_{s^n}, $n \in \mathbb{N}$, for every $n \in \mathbb{N}$ defined on a set of front-worker positions, being homogeneous and monotonic, (ii) an implementation governance (N_n, G_n) with N_n being a non-empty and finite set of positions and G_n being a set of agency relations*

[8] In van den Brink and Ruys (1996) a private firm is modeled which production is valuated at a competitive output price $p > 0$.

[9] For notational convenience we do not consider material cost that depend on the level of production. Considering these costs to have given input price $c > 0$ does not change the results.

[10] Strctural monotonicity implies that the wage offered to a coordinator is always greater or equal to the wages offered to his subordinate workers. Thus, if the workers accept the wages offered to them then also the coordinators accept the wages offered to them. For a hierarchical organization aimed at transmitting information Prat (1997) has shown that the reward system satisfies monotonicity and symmetry.

between these positions having a tree structure with constant span of control, (iii) an agency efficiency parameter $\alpha \in (0,1)$, and (iv) a remuneration system φ satisfying budget neutrality, symmetry, and structural monotonicity.

The set of all n-level firms is denoted by $\mathcal{F}_n$. In this paper we consider a firm as an organization in which the parameter n is a variable. Therefore, a description of a firm should contain for every $n \in \mathbb{N}$ how the firm looks like. The set of positions differs for different $n \in \mathbb{N}$. However the top position i_0 is the same for every size.

Definition 3.2 *A* **firm mapping** *is a function $F: \mathbb{N} \to \bigcup_{n\in\mathbb{N}} \mathcal{F}_n$ such that $F(n) \in \mathcal{F}_n$ for all $n \in \mathbb{N}$.*

The firm has been defined on the set of levels which characterize the governance of the firm in this paper. In principle any parameter of the governance may be chosen. The top-position is identical in any implementation governance, G_n, and determines the number of levels of the structure, n, so as to maximize its profit. Since value added of a firm depends on firm size, the governor can influence value added by choosing the size of the firm, i.e., by fixing the number of hierarchical levels of the firm. Increasing the size of the firm can have a positive and negative effect on the value added. The negative effect results from the level-dependent agency cost as expressed by the parameter $1 - \alpha^n$. The possible positive effect results from the fact that more workers can be active. Since profit of the governor depends on the value added the governor of the firm chooses n in order to maximize profit.

In order for the firm to be active the worker and coordinator positions have to be occupied by employees. We assume the potential employees to have a positive *reservation wage* $w > 0$. They will accept a position in a firm with n levels if and only if the internal wages offered do not fall below their reservation wage w. A firm can only produce if the workers accept the internal wages offered to them. Similarly, the governor has a *reservation rate of return on capital* $r > 0$. If positional returns on capital for the optimal level of the organization are lower than this reservation rate of return on capital, then the governor will not activate the firm, i.e., $n = 0$. Therefore, the governor chooses firm size n such that profit is maximal under the constraint that the wages offered to the workers is at least equal to their reservation wage w. If at this level the positional returns on capital are lower than the reservation rate of return on capital r, then the firm is not activated.

In Definitions 3.1 and 3.2 the internal organization of the firm is described. The *external* organization of the firm is represented by the reservation wage of workers, $w > 0$, and the reservation rate of return on capital of the governor, $r > 0$. In Ruys and van den Brink (1999) the external organization of a firm is specified further by external competitive markets such that the reservation prices w and r are equal to the equilibrium prices on these markets. Here we will take the external reservation prices as given. Denoting $N(w) = \{n \in \mathbb{N} \mid \varphi_n(f_{s^n}) \geq w\}$ and $N(r) = \{n \in \mathbb{N} \mid \varphi_0(f_{s^n}) \geq r\}$ the only firm sizes that are supported by the external environment of the firm are the ones in the set $N(w,r) = N(w) \cap N(r)$, the set of *feasible levels* of the firm. In general, the set $N(w,r)$ can be empty or unbounded.

Definition 3.3 *The* **optimal firm level** *of a firm is the lowest level of the firm that maximizes profit under the constraints that the positional wage of a front-worker position is not lower than the reservation wage, and the positional return on capital is not lower than the reservation rate of return on capital. The function $n: \mathbb{R}^2_+ \to \mathbb{R}$ defined by $n(w,r) = \min\{n \in N(w,r) \mid \varphi_0(f_{s^n}) = \sup_{\hat{n}\in N(w,r)} \varphi_0(f_{\hat{n}})\}$ if $N(w,r) \neq \emptyset$, and $n(w,r) = 0$ if $N(w,r) = \emptyset$ assigns to each pair of positive reservation prices the optimal firm level.*

The optimal firm level is finite if the set $N(w, r)$ is bounded. The following proposition shows that this holds if the value-production functions f_{s^n} are such that average productivity of the labor inputs is non-increasing in firm size n, i.e. $f_{n+1}(s^{n+1}) \leq s f_{s^n}(s^n)$ for all $n \in \mathbb{N}$.

Proposition 3.4 [van den Brink and Ruys (1996)]
Let a firm F with average labor productivity non-increasing in firm size be given. Then, for any positive vector (w, r) of reservation prices, the set $N(w, r)$ of feasible firm levels is bounded.

The above proposition shows that the optimal size of the firm is finite, even for some cases of increasing returns. However, the existence of a positive optimal level is not guaranteed because (N, w, r) might be empty. In the example described in the next section we discuss a class of firms for which the optimal level does exist.

4 Distribution functions

In this section we discuss some distribution functions that can be used as budget-allocation function in a polity or as remuneration system in a firm. In the next section we will then argue how these different distribution functions express different cultural dimensions. The budget-allocation and remuneration system are assumed to be determined before operations start. They may be considered as a system of norms that is culturally determined or agreed upon by the unions.

This section is divided in three subsections. In the first subsection we discuss a particular distribution function, the Banzhaf permission value. In the second subsection we show how this distribution function affects the size of firm and polity governance structures under different complementarities in production and user-value. In the final subsection we generalize the distribution function that is discussed in the first subsection. In Section 5 we then relate the differences between these distribution functions to some cultural dimensions.

4.1 The Banzhaf permission value

In this subsection we discuss a particular distribution function which is based on the game theoretic model with hierarchical permission relations as developed in, e.g., Gilles, Owen and van den Brink (1992), van den Brink and Gilles (1996) and van den Brink (1997). We first describe this distribution function for the firm. It implies that the distribution of the value added among the governor (as profit) and the labor positions (as wages) depends on the value added that can be generated by subsets of worker positions $E \subset W_n$, i.e., all values $v^{f_{s^n}}(k) = \alpha^n f_{s^n}(k)$ for $k \in \{1, \ldots s^n\}$, with $\alpha < 1$. Given an implementation governance (N_n, G_n) the labor positions in $\widehat{G}_n(i) := \{j \in N \mid$ there exists a sequence of positions $(h_1, \ldots, h_t)$ such that $h_1 = i$, $h_{k+1} \in G_n(h_k)$ for all $1 \leq k \leq t-1$ and $h_t = j\}$ are called the *subordinates* of position i, and the positions in $\widehat{G}_n^{-1}(i) := \{j \in N | i \in \widehat{G}_n(j)\}$ are the *superiors* of i. For given size $n \in \mathbb{N}$ we denote by $\sigma_n(E) := \{i \in E \mid \widehat{G}_n^{-1}(i) \subset E\}$ the set positions in E for which all superior positions also belong to E. We refer to this as the *sovereign part* of E.

Now we assume that a front-worker position can only be activated if all its superior coordinator positions are activated. This implies that a firm in which the positions in $E \subset N_n$ are the only ones which are occupied, can only generate the production value that can be produced by the front-worker positions in $\sigma_n(E) \cap W_n$. If the positions $E \subset N_n$ are activated then the *marginal contribution* $m^i_{f_{s^n}}(E)$ of a position $i \in N_n$ to the value added of subset E is the value added that is lost by the firm if the coordinator or worker in position $i \in N_n$ leaves the firm, i.e., $m^i_{f_{s^n}}(E) = v^{f_{s^n}}(|\sigma_n(E) \cap W_n|) - v^{f_{s^n}}(|\sigma_n(E \setminus \{i\}) \cap W_n|)$.

Now the importance of position $i \in N_n$ in the generation of value added can be expressed as the sum of all its marginal contributions over all coalitions, i.e., by $\mu^B_i(f_{s^n}) = \sum_{\substack{E \subset N_n \\ E \ni i}} m_i^{f_{s^n}}(E)$. Although the values $\mu^B_i(f_{s^n})$ obtained in this way satisfy symmetry and structural monotonicity they need not be budget neutral[11]. In order to get budget neutrality (without losing symmetry and structural monotonicity) we can distribute the value added of the fully employed firm proportional to these payoffs yielding the *Banzhaf permission value*[12], φ^B.

Similarly the Banzhaf permission value β^B for a polity can be defined. However, then the user-value generated by members should be reallocated as budgets to agents at higher levels (instead of to principals as done in the firm). So, for given size $n \in \mathbb{N}$ we denote by $\sigma^c_n(E) := \{i \in E \mid \widehat{G}_n(i) \subset E\}$ the *sovereign part* of E, and the marginal contribution of a position $i \in N_n$ to the value taxed of a subset $E \subset N_n$ in the polity is given by $v^n(|\sigma^c_n(E) \cap N_0|) - v^n(|\sigma^c_n(E \setminus \{i\}) \cap N_0|)$.

4.2 Complementarity in value-production and in user-value

Before generalizing the Banzhaf permission value we show some properties of the Banzhaf permission value for firms and clubs with different degrees of complementarity in production and user-value expressed by different constant elasticity of substitution (CES) value-production and user-value functions.

Although the value-production of a firm is different for different sizes n we assume that the structural technology characteristics are invariant under size. For the firm one extreme case is that of a linear production technology (with substitutable labor inputs) given by the value-production function $f^1_{s^n}(k) = k$ for all $n \in \mathbb{N}$. The corresponding value added for $k \in \{0, 1, \ldots, s^n\}$ front-worker positions activated is given by $v^{f^1_n}(k) = \alpha^n k$. Profit and front worker position wages are given by $\varphi^B_0(f^1_{s^n}) = \frac{(\alpha s)^n}{n+1}$, and $\varphi^B_n(f^1_{s^n}) = \frac{\alpha^n}{n+1}$, respectively.

The wage assigned to the front-worker positions is decreasing with firm size n, while for reasonable values[13] of α profit is increasing[14] in n. In this case the set $N(w)$ is connected and bounded from above by the *reservation-wage level*, n_w, being the level of coordination above which the wage of workers is lower than their reservation wage, w. The firm is inactive if $\varphi^S_0(f^1_{n_w}) < r$ or $\varphi^S_n(f^1_s) < w$. If activated, the governor will choose the deepest organization structure restricted by the reservation wage of the workers, n_w.

At the other extreme, for a Cobb-Douglas value-production function (with indispensible labor inputs) given by[15] $f^0_{s^n}(s^n) = s^n$ and $f^0_{s^n}(k) = 0$ if $k < s^n$ for all $n \in \mathbb{N}$, value added of a firm

[11]Multyplying $\mu^B(f_{s^n})$ by $\frac{1}{2^{|N_n|-1}}$ yields a remuneration system that can be obtained as the *Banzhaf value* (Banzhaf (1965)) of a corresponding TU-game. Still budget neutrality is not guaranteed.

[12]This value can be obtained as the *normalized Banzhaf value* of a corresponding TU-game as characterized by van den Brink and van der Laan (1998).

[13]For $s = 2$ φ_0 is increasing in n (for $n \geq 1$) for $\alpha \geq \frac{e^{\frac{1}{2}}}{2} \simeq 0.825$. For $s > 2$ this is even the case for lower values of α. Williamson (1967) argues that α mostly will be in the neighborhood of 0.9.

[14]Otherwise, profit is decreasing to a minimum, and increasing monotonically from that level.

[15]We normalize production in a way such that total value production of a fully employed firm is the same as for the linear production firm. So, the firms only differ with respect to the substitutability of the labor inputs.

with $k \in \{0, 1, \ldots, s^n\}$ active front-worker positions is equal to $v^{f^0_{s^n}}(k) = (\alpha s)^n$ if $k = s^n$, and equal to 0 otherwise. Profit of the governor and wages for the front-worker positions are given by $\varphi^B_0(f^0_{s^n}) = \varphi^B_n(f^0_{s^n}) = \frac{(\alpha s)^n}{\sum_{k=0}^n s^k} = \frac{(\alpha s)^n (s-1)}{(s^{n+1}-1)}$, and thus are both decreasing in the number of hierarchical levels. It follows that the governor sets firm size n not higher than 1, the flattest possible structure. If the working labor inputs are indispensable then the firm is inactive if $\varphi^B_0(f^0_s) < r$ or $\varphi^S_n(f^0_s) < w$. Otherwise the governor will choose the flattest organization $n = 1$, and wages will equal profit.

So, for reasonable values of α, a linear production technology (with substitutable labor inputs) yields a firm with a deep implementation governance, while a Cobb-Douglas production technology (with indispensable labor inputs) yields a firm with a flat implementation governance. For intermediate cases the size of the governance can be between these two extremes.

Example 4.1 Consider two firms with $s = 2$, and $\alpha = 0.7$. Table 2 gives the corresponding profits and wages for the linear and Cobb-Douglas firm.

n	$\varphi^B_0(f^1_{s^n})$	$\varphi^B_n(f^1_{s^n})$	$\varphi^B_0(f^0_{s^n}) = \varphi^B_n(f^0_{s^n})$
1	0.700	0.350	0.467
2	0.653	0.163	0.280
3	0.686	0.086	0.183
4	0.768	0.048	0.124
5	0.896	0.028	0.085

Table 1: $s = 2, \ \alpha = 0.7$

For the linear production firm profit is minimal for $\breve{n} = 2$. If, $w = 0.03$ then the governors of the firm will push the workers to their reservation wages and set firm size equal to 4 (if $r \leq 0.768$).

If $w, r \leq 0.467$ the firm with the Cobb-Douglas technology will have one hierarchical level. Otherwise it will not be active.

□

In a similar way the Banzhaf permission value can be used in allocating the budgets to positions in a polity, resulting from the user-value of subsets of members as given by the user-value function u. The sum of budgets to be assigned to the positions in an n-level polity equals the total user-value $v^n(N_n) = \sum_{P \in \mathcal{P}^n} u(N_0 \cap P)$.

Applying the Banzhaf permission value as a budget allocation function β^B and considering various degrees of complementarity we come to the following conclusions. If user-value is *separable*, as represented by the linear user-value function $u(E) = |E|$ for all $E \subset N_0$, then the budget assigned to the members is given by $\beta^B_0(u, n) = \frac{1}{n+1}$, and thus is decreasing in size n. The optimal polity size then equals $n^* = 0$. Members do not want to pay taxes to invest in a representation governance because governance cannot improve user-value, while it induces the cost of the officer positions. (This illustrates the first part of the result stated in Proposition 2.4 for more general budget-allocation functions.)

The other extreme is a complementary user-value function given by $u(N_0) = |N_0|$, and $u(E) = 0$ if $E \neq N_0$. In this case $\beta_0^B(u, n_{max}) = \frac{|E|}{|N_{n_{max}}|}$, and $\beta_0^B(u, n) = 0$ if $n < n_{max}$. Full governance is necessary in order to generate a positive user-value. Optimal polity size $n^* = n_{max}$, and thus the maximal number of governance levels will be formed. This is the only structure that yields a positive user-value. (This illustrates the second part of the result stated in Proposition 2.4 for more general budget-allocation functions.) For intermediate cases governance levels between 0 and n_{max} can be possible.

Example 4.2 Consider a polity with $m = 4$ members $N_0 = \{i_{0,1}, i_{0,2}, i_{0,3}, i_{0,4}\}$ and scope of user-interaction $s = 2$. The three possible representation governance structures are the two illustrated in Figure 1 and the 'empty' structure in which there is no governance. If the user-value function is linear then $n^* = 0$. In case of a complementary user-value function $n^* = n_{max} = {}^s\log m = 2$. □

4.3 Alternative distribution functions

In this subsection we present a generalization of the Banzhaf permission value that contains classes of distribution functions that will be related to cultural dimensions in the next section. Again we first define the generalization in terms of the firm. The Banzhaf permission value is obtained by distributing the value added of the firm proportional to the sums of marginal contributions $\mu_i^B(f_{s^n})$ over the positions $i \in N_n$. Instead of taking the sum of marginal contributions (in which each coalition size gets the same weight) we can assign possibly different weights to coalitions of different size. So, instead of measuring the importance of positions by $\mu^B(f_{s^n})$ we can measure it by

$$\mu_i^\omega(f_{s^n}) = \sum_{\substack{E \subset N_n \\ E \ni i}} \omega_{|E|} \cdot m_i^{f_{s^n}}(E),$$

with weights $\omega_t > 0$ for $t \in \{1, \ldots, |N_n|n\}$. These weights express the importance put on the marginal contributions to coalitions of positions of different size. Following van der Laan and van den Brink (1998) we then obtain budget neutral distribution functions by distributing the value added of a firm proportional to the values $\mu_i^\omega(f_{s^n})$, i.e., we consider distribution functions θ^ω given by

$$\theta_i^\omega(f_{s^n}) = \frac{\mu_i^\omega(f_{s^n})}{\sum_{j \in N_n} \mu_j^\omega(f_{s^n})} v^{f_{s^n}}(s^n) \text{ for all } i \in N_n. \tag{1}$$

By Θ we denote the class of all distribution functions that can be obtained in this way. Special cases of such distribution functions are the Banzhaf permission value which is obtained be taking equal weights for all coalition sizes, or the *Shapley permission value* used in van den Brink (1996) and van den Brink and Ruys (1996), obtained by taking the Shapley weights[16] $\omega_t^S := \frac{(|N_n|-t)!(t-1)!}{|N_n|!}$ for all $t \in \{1, \ldots, |N_n|\}$.

All distribution functions in Θ satisfy budget neutrality, symmetry and structural monotonicity, and thus can be used as a remuneration system in a firm. (Since they also satisfy the non-negativity condition required for budget-allocation functions in Proposition 2.4 they also can be used as such functions in a polity.)

[16] This value can be obtained as the *Shapley value* (Shapley (1953)) of the corresponding TU-game.

Proposition 4.3 *Every distribution function $\theta^\omega \in \Theta$ satisfies budget neutrality, symmetry, and structural monotonicity.*

PROOF
(We will use the notation for the firm introduced in the previous section. For the polity the same holds with adapting notation.)
Budget neutrality of θ^ω follows from (1) and the fact that $\mu_i^\omega(f_{s^n}) > 0$ for all $i \in N_n$ if if $v^{f_{s^n}}(s^n) > 0$.
Symmetry follows from the fact that coalitions of the same size are assigned equal weight.
Finally, structural monotonicity follows from the definition of the marginal contributions and the facts that (i) for $j \in G_n(i)$ it holds that $j \in \sigma_n(E)$ implies that $i \in \sigma_n(E)$, and (ii) $\sigma_n(E \setminus \{i\}) \subset \sigma_n(E)$ for all $E \subset N_n$ and $i \in E$.

□

From this proposition it follows that Propositions 2.4 and 3.4 are valid if we take any of the distribution functions in Θ as budget-allocation function in the polity or remuneration system in the firm.

5 Cultural dimensions in governance

Clubs and firms operate in an external environment that is represented by several parameters determining the internal performance of these organizations. The prices of marketable goods are standard parameters in a market economy. New parameters are determined not only by considerations of economic rationality but also by the specific features of societies, in particular their cultural characteristics. In addition, culture may affect the objective function which is maximized in choosing the governance structures. In these two ways, the culture of a society will influence the governance structures of organizations operating in this society. In his seminal work, Hofstede (1980) identified and introduced some dimensions of culture that characterize a society, viz. its governance structures. These characteristics have been described intuitively. Our approach allows for a precise decription of these cultural dimensions, although this precise description may exclude some intuitions that would otherwise be admitted.

Consider a firm implementation governance. The utility functions and participation constraints of workers are crucial for determining their remuneration system and other aspects of the implementation governance. These functions and constraints vary between cultures. Sure enough, current wages will always be one of their components, but it may be weighted with other factors; job security and work atmosphere are just two examples. Workers may have reservation levels of job security and other factors as elements of their participation constraint. Workers may be motivated by other factors than their wages. So why would the firm's governor agree with any additional requirements on the workers' part? If such requirements are the components of participation constraints of workers, for cultural reasons, the governors may have no choice. They also may have little choice if taking such requirements into account is dictated by strong social norms or government regulations (and such norms and regulations would be caused by corresponding cultural factors). Alternatively, the firm may discover that satisfying such requirements leads to a greater loyalty and motivation of workers, which increases workers' effort or allows to introduce more efficient technologies which require such an attitude from workers (such as diversified quality production in Germany and 'lean' production in Japan). We suggest that cultural factors in a society influence the representation governance, as well as the implementation governance in that society.

Uncertainty avoidance refers to the extent to which people feel threatened by uncertainty and ambiguity and try to avoid these situations. People in societies with different uncertainty avoidance will differ in the extent to which certainty and stability in their life is important for them, and thus in the extent they are prepared to give up other goods in exchange for such a certainty. For workers, in particular, the desire for stability in life will manifest itself in the desire for a higher stability of income, that is, for a better job security.

In a society with low uncertainty avoidance, privatization is easy to accomplish. People are certain of the delivery of separated activities. Some production activities may be very vulnarable because imperfectly monitoring markets are not able to correctly asses the value added, e.g., non-profit services, or because the international environment is unstable. They only can be continued if the cost of producing is partly borne by the workers. In a high uncertainty avoiding society the front-workers prefer to continue the production of complex goods with a lower externally measured value and to accept a lower internal wage, rather than changing position. So the internal effect of external uncertainty focusses on the internal remuneration system of the firm.

In case of a high external uncertainty front-workers are supposed to be willing to accept low wages and to abstain from positional rents in order to pay for the cost of the relative strength, the independence or the continuity of the firm. Residual rents flow to the top-position and an ex-ante fixed wage system for the other positions is part of the remuneration system of the whole firm. Assuming that this attitude is a structural one, it causes an unequal income distribution within the firm and in society on the long term or in expectation.

In the governance model described in this paper uncertainty avoidance in a firm can be expressed as follows. For the remuneration in a firm, we consider a generalization of the distribution functions in the class Θ discussed in the previous section by taking convex combinations of any of these functions and the *fixed wage schedule* which coincides with the wages used by Williamson (1967). In the fixed wage schedule wages assigned to front-worker positions, denoted by w_n, and the ratio between the wage of a coordinator position (except the governor's position) and its subordinate labor positions, denoted by γ, is fixed and greater than one. This implies that all wages are fixed and independent of value added. The profit of the governor then is obtained as a residual profit by subtracting all labor cost from the value added. If $w_k = \gamma^{n-k} w_n$, $k = 1, \ldots, n$, is the wage assigned to employee positions in level k, then profit equals $\pi(f_{s^n}) = v^{f_{s^n}}(s^n) - \sum_{k=1}^{n} s^k w_k$. In order to express various degrees of uncertainty avoidance we take convex combinations

$$\theta^{\psi} = \psi \overline{w} + (1-\psi)\theta \text{ for all } \psi \in [0,1]$$

where $\theta \in \Theta$ and $\overline{w} \in \mathbb{R}^n$ is the fixed wage schedule. In θ the rewards assigned to all positions depend on value added, and $\overline{w}$ is a fixed wage schedule in which the rewards assigned to all labor positions are fixed and profit equals value added minus the sum of all wages. In the model described in this paper value added is deterministic and therefore also profit is deterministic. Uncertainty could enter the model in the following way. The fixed way schedule $\overline{w}$ is a stochastic variable which probability distribution is known. So, a worker (or other potential employee) who wants to occupy a particular position in a firm does not know the wage it eventually will earn, but does know its expected wage. The final wage outcome might, for example, depend on negotiations between the employee and the employer (or, if centralized, between labor unions and employer organizations) or government regulation. Besides this, the wages according to $\theta \in \Theta$ also depend on the outcome of value added. However, in an uncertain environment, value added is the outcome of a complex system of external and internal factors such as the functioning of markets (yielding prices) and the internal production processes between groups of employees. So, in general the probability distribution of value added is unknown, and thus wages that depend on value added (such as the wages in Θ) are uncertain. Therefore, the weight put on the fixed wage schedule can be seen as a measure of uncertainty avoidance in a firm[17].

Definition 5.1 *Uncertainty avoidance in a firm*
The internal effect on a firm caused by the uncertainty about the performance of external institutions is called **uncertainty avoidance**. *It is measured by the weight (between zero and one) put on the fixed wage schedule in the composition of the remuneration system used. Uncertainty avoidance is* **low** *if this weight is close to zero and* **high** *if it is close to one.*

For arbitrary weight ψ, Proposition 3.4 need not be satisfied. The fixed wage schedule satisfies budget neutrality and symmetry. It satisfies structural monotonicity for the labor positions, but clearly the profit of the governor position can be lower than wages. If $\psi w_n < w$ (where w denotes the reservation wage) in the limit as n goes to infinity, the wage assigned to front-worker positions falls below the reservation wage, since $\theta_n(f_{s^n})$ will go to 0 if n goes to infinity, and thus $N(w)$ is bounded in that case. If $\psi w_n > w$ then wage will never fall below the reservation wage and $N(w)$ is not bounded. For $\psi = 1$, i.e, putting full weight on the fixed wage schedule, it holds that $\overline{N}$ is bounded since $N(r)$ is bounded. For the linear production case this is shown by Williamson (1967). For other value-production functions this follows in a similar way.

Before discussing the influence of collectivism on a firm we shall analyse its influence on a club. In some (individualist) societies people can entrust other people with organizing the provision of collective goods on the basis of a contract. In these societies there is no restriction on the formation of clubs; collective goods can be provided via clubs in which the members do not have anything else in common than an interest in this particular collective good. In other (collectivist) societies, the contract would be not enough: a trust relationship has to be established before such a delegation. Such a relationship can be based on existing meaningful association between people (family ties, common religion, race etc.) or have to be established anew. The degree of trust necessary may differ for different purposes. Sometimes a trust that is based on common association is sufficient, but often additional investments in establishing trust are necessary. As a result, in collectivist societies people prefer to organize the provision of collective goods, and of many private goods, within specific groups of people who are associated with each other in some way (besides having a common interest in a collective good), e.g. through family ties, common religion, race, or working at the same enterprise. This factor creates a restriction on the set of clubs that can be formed. One of the most important aspects of such a restriction is that in most cases there would be an exogenous limit on the size of the club that could be formed. If the trust is based on the previous common association, only a fraction of society's members can join the club. If the

[17] The uncertainty described here is often called *ambiguity* or *Knightian uncertainty* by economists, see, e.g., Schmeidler (1989), Gilboa and Schmeidler (1989) and Fishburn (1993).

trust is established anew, for the most purposes there would be a limit on the size of collective in which trust can be effectively established and maintained. The effect of such a restriction on the governance system is an indication of the degree of collectivism.

The implications of collectivism to the firm can be considered in a similar way. In collectivist societies people prefer to have 'family-like' trust relationships within a firm, and are not satisfied simply with business-like relationships of exchange of labor for wages. This may impose a restriction on the size of a firm which can be formed (alternatively, if trust relationships are not an absolute necessity but enhance productivity, this may pose a limit on the size of a firm that can be efficient)

A specific group of members which induces a trust-relation which delegates power to higher echelons is called a *clan*. The degree of collectivism-individualism can be expressed as the importance of clan-size in a special class of distribution functions in Θ. This class consists of those distribution functions that can be obtained according to (1) with weights given by

$$\omega_t = \begin{cases} c + \frac{t-1}{n^c-1}(c^* - c) & \text{if } 1 \leq t \leq n^c \\ \\ c + \frac{\overline{n}-t}{\overline{n}-n^c}(c^* - c) & \text{if } n^c < t \leq \overline{n} \end{cases} \tag{2}$$

for some firm size $n^c \in \mathbb{N}$ and numbers $c, c^* \in \mathbb{R}_+$, with $\overline{n} = |N_n|$. These weights assign a maximal weight c^* to a particular coalition size n^c, a weight c to the largest and smallest coalition size (1 and $\overline{n}$, respectively), and the weights of other coalition sizes are determined by a linear relation. The coalition size n^c which gets the highest weight thus is the most important in the determination of the distribution of value added or budgets, and is refered to as the *clan size*. The weight assigned to this clan size compared to weights assigned to other sizes can be seen as an indicator of the importance of clan size, and thus as a measure of collectivism. This can be measured by the average of the weights assigned to all coalition sizes divided by the weight assigned to the clan size. This number lies between zero and one. The lower this number, the more important is clan size, and thus the more collectivist is the organization.

Definition 5.2 *Relations in a club and a firm*
The members in a society forming clubs may be partitioned according to a specific characteristic in a set of coalitions, each called a **clan**, *within which the members are inclined to delegate power to a commissioner. This cultural clan-structure puts a restriction on club formation. Its effect is measured by the ratio κ of the average weight assigned to all coalition sizes to the weight assigned to the clan size (between zero and one). The higher the relative weight assigned to clan size (and thus the lower the ratio κ), the more collectivistic is the club. A club is called* **collectivistic** *if the ratio is close to zero. It is called* **individualistic** *if the clan-restriction is absent and κ is equal to one. This definition also applies to the firm.*

The most individualistic distribution functions assign equal weight to all sizes, and thus clan size is not important. It is easy to verify that the Banzhaf permission value is obtained by taking $c = c^*$ and so clan size n^c does not matter. Thus, the Banzhaf permission value can be seen as an individualistic distribution function.

The most collectivist distribution functions are the ones for which the clan size is assigned a positive weight, and all other sizes are assigned weight zero. An example of such a distribution function is the one which puts full weight on the largest coalition size[18], i.e., for which

[18] This yields a distribution function that is related to the τ-value for TU-games (see Tijs (1981)).

$n^c = \overline{n}$, $c = 0$ and $c^* > 0$. Another example is obtained by taking $n^c = 1, c^* = 0$ and $c > 0$ which puts full weight on the coalitions consisting of individual positions[19].

These concepts describe and characterize aspects of a governance[20]. Proxies of these concepts have been measured empirically by Hofstede (1980). It is important, however, to know whether such a characteristic is inevitable and technologically determined, or whether it represents a real identity of the society because it is freely chosen and not determined by technological conditions. An answer on this question can be given by our approach. The difference observed between the actual and the feasible characteristics determines the cultural identity of a society.

6 Conclusion

In this paper a neo-institutional governance structure has been presented for the procurement and the provision of a common service. The provision takes place in firms with an implementation governance, forming the industry of that common service. The procurement is organized by clubs with a representation governance, forming the polity of the common service. Clubs empower firms, as the legislative branch of a government empowers the executive branch. An optimal governance for an industry and for a polity can be derived by maximizing its objective functions, resulting in standard characteristics of a governance. Simultaneously cultural dimensions of a society can be defined precisely in terms of these governance characteristics. We have shown that cultural values existing in a society influence governance. This may lead to actual governance systems that deviate in some ways from the standard governance. A government policy of changing the actual, culturally influenced governance in the direction of the standard, optimal governance goes at a substantial cost, what we call *social transition cost.* A policy of not imposing the standard governance, however, will cause another type of cost, which may be called *social transaction cost.* The fundamental questions are whether, how fast and how far should a society aim at implementing the standard governance. Or should the government guard the society's cultural identity and is the society prepared to pay for it, given the associated social transaction cost? Although these questions are not answered, tools are presented here which may contribute to formulating the questions more precisely.

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[19] Although the weights in these collectivistic distribution functions are not all positive, Proposition 4.3 also holds for these distribution functions if the value-production, respectively, user-value function is supermodular.

[20] Two other cultural dimensions also could be made precise within the model. *Power distance* refers to the extent to which the less powerful people accept the fact that power is distributed unequally. *Femininity-masculinity* deals with a relative emphasis in society on achievement and success, on the one hand, and caring for others and quality of life, on the other hand.

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